A CONTACT METHOD OF DETERMINATION OF THERMOPHYSICAL PROPERTIES OF ROCKS FROM CORE SAMPLES

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The zone of the action of thermal disturbances around a circular heat source on the surface of a semi-infinite body is estimated with the aim of using contact methods of determination of thermophysical properties of materials from core samples.

Recently, contact methods of determination of thermophysical properties of materials are more and more extensively used in measuring practice. They are based on the behavior of the temperature field of a semi-infinite body under the action of a local heat source on its surface. Numerous variants of these methods, which differ in the scheme of account for or elimination of heat losses into the environment, and the time of measurement of the temperature of the body in the initial or final stage of experiments, have been proposed [1-12]. Methods based on measurement of temperature in the initial stage of heating of the material (Fo < 0.1) are of interest [10] for a short experiment time. However, in this case the heat capacity of the heater distorts the temperature field of the body, which leads to additional experimental errors. This undesirable effect can be eliminated if the stage of steady-state linear dependence of temperature on the parameter $1/\sqrt{\tau}$ is chosen as the final stage of measurements [1, 12]. However, in geothermal studies of deep horizons of the Earth's crust using core samples, boundary effects can be observed in the temperature field of the sample, because of its limited size. Therefore, studies are required to estimate the zone of action of temperature disturbances around the heat source and to choose the necessary size of the heat source for the experiment time with heat losses to the environment taken into consideration.

The calculations are based on solution of the problem on the temperature field of a semi-infinite body the surface of which has a heat source with a constant heat flux q and radius R [13]:

$$\vartheta(r, z, \tau) = \frac{qR}{2\lambda} \int_0^\infty J_0(\beta r) J_1(\beta R) \times \left[\exp(-\beta z) \operatorname{erfc}\left(\frac{z}{2\sqrt{a\tau}} - \beta\sqrt{a\tau} \right) - \right.$$
$$\left. - \exp(\beta z) \operatorname{erfc}\left(\frac{z}{2\sqrt{a\tau}} + \beta\sqrt{a\tau} \right) \right] \frac{d\beta}{\beta}, \qquad (1)$$

where J_0 and J_1 are first-kind Bessel functions of the zeroth and first order; erfc = 1−erf; erf, is the Gaussian error function.

It is difficult to express formula (1) in explicit form. In practice, it is sufficient to study the temperature distribution along the central axes Oz and Or.

At large times when $z/2\sqrt{a\tau} < 1$ for the central axis $Oz(r = 0)$ Eq. (1) is reduced to the following series

$$\vartheta(0, z, \tau) = \frac{q}{\lambda} \left\{ \sqrt{z^2 + R^2} - z - \frac{1}{2\sqrt{\pi a\tau}} \times \right.$$
$$\left. \times \left[R^2 - \frac{(z^2 + R^2)^2 - z^4}{24a\tau} + \frac{(z^2 + R^2)^3 - z^6}{480a^2\tau^2} - \dots \right] \right\}. \qquad (2)$$

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After a certain time (τ_{lin}) of action of the heat source the terms containing a_τ in brackets in Eq. (2) are negligible in comparison with those in Eq. (1) and the temperature field becomes a linear asymptote in the parameter $1/\sqrt{t}$:

$$\vartheta(0, z, \tau) = \frac{qR}{\lambda}\left[A_{st}(z, 0) - \frac{R}{2\sqrt{\pi a \tau}}\right], \tag{3}$$

where $A_{st}(z, 0) = (\sqrt{z^2+R^2} - z)/R$ is the steady-state distribution of the temperature in z at $\tau \to \infty$. Thus, under asymptotic conditions the temperature field can be expressed by a series of parallel straight lines $\vartheta \to f(1/\sqrt{\tau})$. The moment at which the temperature curves $\vartheta \to f(1/\sqrt{\tau})$ become a linear asymptote depends on z and is expressed by the following relation:

$$\tau_{lin} = \frac{R^2}{24a\varepsilon_{lin}}\left(\frac{2z^2}{R^2} + 1\right), \tag{4}$$

where ε_{lin} is the permissible error of deviation of the temperature curve $\vartheta \to f(1/\sqrt{\tau})$ from linearity.

In a similar way it can be shown that in the central plane $z = 0$, where the heat source is located, the radial temperature distribution tends with time to the following linear asymptote

$$\vartheta(r, 0, \tau) = \frac{qR}{\lambda}\left[A_{st}(r, 0) - \frac{R}{2\sqrt{\pi a \tau}}\right], \tag{5}$$

where $A_{st}(r, 0) = \int_0^\infty J_0(\beta r)J_1(\beta R)d\beta/\beta$ is the steady-state temperature distribution in the radial direction in the plane of action of a circular heat source ($z = 0$).

Integrals of the type of $A_{st}(r, 0)$ are expressed by a combination of gamma (G)- and hypergeometric (F) functions:

in the region $r \le R$

$$A_{st}(r, 0) = F\left(\frac{1}{2}; -\frac{1}{2}; 1; \frac{r^2}{R^2}\right) = 1 - \frac{r^2}{4R^2} - \frac{3r^4}{64R^4} - \frac{5r^6}{256R^6} - \ldots, \tag{6}$$

in the region $r \ge R$

$$A_{st}(r, 0) = \frac{R}{2r}F\left(\frac{1}{2}; \frac{1}{2}; 2; \frac{R^2}{r^2}\right) = \frac{R}{2r}\left(1 + \frac{R^2}{8r^2} + \frac{3R^4}{64r^4} + \frac{25R^6}{1024r^6} + \ldots\right). \tag{7}$$

For realization of Eq. (5) a method of a plane probe is suggested for measuring thermophysical properties of a mass of permafrost ground and rock [7, 12]. Its essence can be described as follows. For some time the temperature of the probe is measured, and these measurements are used to plot ϑ versus the parameter $1/\sqrt{\tau}$, and the linear asymptote is isolated in the plot. From the intersection points of the asymptote with the coordinate axes it is possible to find the thermal conductivity and thermal diffusivity of the material. In particular,

$$\lambda = \frac{8qR}{3\pi\vartheta_{st}} - \lambda_{ins}, \tag{8}$$

where ϑ_{st} is the steady-state increase in the average temperature of the probe determined by the intersection of the linear asymptote with the ordinate; λ_{ins} is the thermal conductivity of the thermal insulation of the instrument.

For the case of samples of finite dimensions, we will investigate their minimal admissible dimensions, which ensure reliable operation of the method described above. For this purpose we will estimate the zone of action of the heat source, both along the radius (r_{act}) and the height (z_{act}), beyond which during the experiment the temperature disturbance relative to a steady-state increase in the temperature of the center of the heater $\vartheta_{st}(0, 0)$ does not exceed the admissible error ε_{act}:

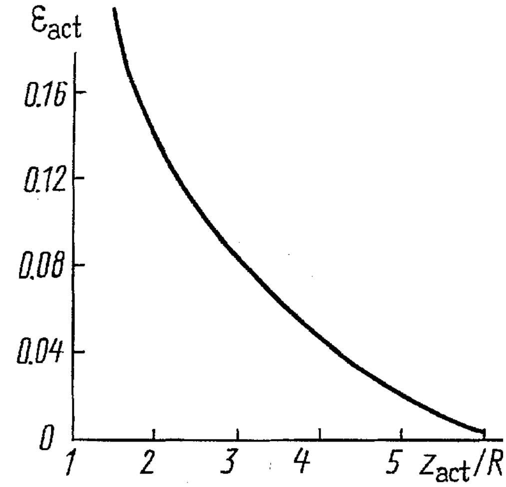

Fig. 1. z_{act}/R vs. ε_{act} at $\varepsilon_{lin} = 0.05$ and $n = 10$.

$$\varepsilon_{act} = \frac{\vartheta(r_{act}, 0, \tau_{exp})}{\vartheta_{st}(0, 0)} = \frac{\vartheta(0, z_{act}, \tau_{exp})}{\vartheta_{st}(0, 0)}, \tag{9}$$

where $\tau_{exp} = \tau_{lin} + \Delta\tau$ is the duration of the experiment; τ is the time of the linear section of the temperature curve in the parameter $1/\sqrt{\tau}$ sufficient for linear asymptote to be drawn reliably over the experimental points in the plot $\vartheta \to f(1/\sqrt{\tau})$.

Formula (9) expreass a strict experimental condition. In the case of a semi-infinite body the theory of the method does not assume any restrictions for the temperature disturbance around the heater. When the body studied has finite dimensions any increase in the temperature at the interface will bring about distortion of the behavior of the temperature at the point of measurement.

Calculation of $\vartheta(z_{act}, 0, \tau_{exp})$ according to Eq. (2) gives

$$\varepsilon_{act} = \sqrt{\left(1 + \frac{z_{act}^2}{R^2}\right)} - \frac{z_{act}}{R} - \frac{r}{2\sqrt{\pi a \tau_{exp}}} \times \left\{1 - \frac{R^2}{24 a \tau_{exp}}\left(1 + \frac{2z_{act}^2}{R^2}\right) + \frac{R^4}{480 a^2 \tau_{exp}^2} \times \right.$$

$$\left. \times \left(1 + \frac{3z_{act}^2}{R^2} + \frac{3z_{act}^4}{R^4}\right) - \ldots\right\}. \tag{10}$$

Hence, with the radius of the heater R, thermal diffusivity of the sample a, duration of the experiment τ_{exp}, and admissible deviation of the temperature ε_{act} preset, the necessary dimension of the sample $l_{sam} = z_{act}$ can be estimated. With condition (9) satisfied the theory of the method [12] is not actually violated and the errors in determination of the thermophysical properties of a rock sample are only caused by instrumental errors of measuring the parameters in the calculation formula, namely, temperature, time, and heater current. When the theory is violated, the plot of ϑ versus $1/\sqrt{\tau}$ shows deviation from linearity.

The following example will be considered for illustration. Let the duration of the experiment be n times longer than the time τ_{lin} of the start of the linear section in the plot of ϑ versus $1/\sqrt{\tau}$. It is assumed that this ensures that the linear asymptote be drawn over the experimental points. For this time the change in the temperature on the boundary of the sample ($l_{sam} = z_{act}$) should not exceed the instrumental error in measurement of ε_{act}. We have $\tau_{exp} = n\tau_{lin} = nR^2/24a\varepsilon_{lin}$. Then,

$$\varepsilon_{act} = \sqrt{\left(1 + \frac{z_{act}^2}{R^2}\right)} - \frac{z_{act}}{R} - \frac{1.38\sqrt{\varepsilon_{lin}}}{\sqrt{n}} \times$$

$$\times \left\{ 1 - \frac{\varepsilon_{lin}}{n} \left(1 + \frac{2z_{act}^2}{R^2} \right) + \frac{1.2\varepsilon_{lin}^2}{n^2} \left(1 + \frac{3z_{act}^2}{R^2} + \frac{3z_{act}^4}{R^4} \right) - \ldots \right\}. \quad (11)$$

This relation is shown in Fig. 1 for $n = 10$ and $\varepsilon_{lin} = 0.05$. If we take only $\varepsilon_{act} = 0.02$, then it should be that $z_{act}/R \geq 5$ or $l_{sam} \geq 5R$. For example, with a heater radius of 0.5 cm, the height of the sample should be at least 2.5 cm for experiment time $\tau_{exp} \geq 10\tau_{lin}$. It can be expected that this is also valid for r_{act}/R.

A maximum zone of action is achieved under steady-state thermal conditions ($\tau \to \infty$):

$$\left(\frac{z_{act}}{R} \right)_{max} = \frac{1 + \varepsilon_{act}^2}{2\varepsilon_{act}}, \quad (12)$$

$$\left(\frac{r_{act}}{R} \right)_{max} = \frac{1}{2\varepsilon_{act}}. \quad (13)$$

For example, at $\varepsilon_{act} = 0.02$, we have $(z_{act})_{max} = (r_{act})_{max} = 25R$.

Now, we will study the heat losses from the heater through the thermal insulation of the device via the leads of the thermocouple and the heater. The estimation is based on the maximum heat loss that occurs under steady-state thermal conditions and is equal to

$$q_{h.l} = q_{ins} + q_{ld} = q_{ins} + \frac{\lambda_{ld} S_{ld} \vartheta_{st}(0,\ 0)}{l_{ld} S_h}, \quad (14)$$

where $\vartheta_{st}(0, 0)$ is the steady-state increase in the temperature of the heater; S_h and S_{ld} are the surface areas of the heater and the cross-section of the leads, respectively; the subscripts "ins" and "ld" refer to the thermal insulation and leads.

In contact heating of a two-layer medium (thermal insulation + the body studied), the following relations are valid

$$\vartheta_{st}(0,\ 0) = \frac{qR}{\lambda + \lambda_{ins}}, \quad (15)$$

$$q_{ins} = \frac{q\lambda_{ins}}{\lambda + \lambda_{ins}}. \quad (16)$$

With account of expressions (15) and (16), we obtain from Eq. (14)

$$q_{h.l} = \frac{q}{(\lambda + \lambda_{ins})} \left(\lambda_{ins} + \frac{\lambda_{ld} R_{ld}^2}{l_{ld} R} \right). \quad (17)$$

It is necessary that in comparison with the heat flux of the heater, the heat loss not exceed the admissible error $\varepsilon_{h.l}$: $q_{h.l}/q \leq \varepsilon_{h.l}$. Then,

$$\varepsilon_{ins} + \varepsilon_{ld} \leq \varepsilon_{h.l}, \quad (18)$$

where $\varepsilon_{ins} = \lambda_{ins}/\lambda_{ins} + \lambda$ and $\varepsilon_{ld} = \lambda_{ld} R_{ld}^2/(\lambda_{ins} + \lambda) l_{ld} R$ are the fractions of the heat losses from the heater through the thermal insulation and leads, respectively.

Thus, it is found that the optimal experimental conditions are determined by the dimensions of the heater. For example, with a heater with small dimensions, the necessary (linear) heating starts more rapidly, the experimental time is reduced, the zone of action of thermal disturbances of the heat source becomes more narrow, the boundary effects due to a limited size of the sample are minimized, and distortion of the temperature field of the sample is decreased. However, the dimensions of the heater cannot be too small, since in this case it will only generate heat for heat losses via the leads of the device.

Therefore, it is necessary to determine the minimum permissible dimensions of the heater more accurately. They can be determined from Eq. (18) on the basis of the preset value of ε_{ld}:

$$R_{min} \geq \frac{\lambda_{ld} R_{ld}^2}{\varepsilon_{ld} l_{ld} (\lambda_{ins} + \lambda)}, \tag{19}$$

where $\varepsilon_{ld} = \varepsilon_{h.l} - \varepsilon_{ins}$.

The results of the present studies can be useful in developing express methods for measurement of thermophysical properties of rock samples and other materials.

NOTATION

r, z, instantaneous coordinates; τ, time; $T(r, z, \tau)$, temperature distribution of a semi-infinite body in space at any time; T_0, initial temperature of the body; $\vartheta(r, z, \tau) = T(r, z, \tau) - T_0$, excessive temperature of the body; λ, thermal conductivity; a, thermal diffusivity; q, heat flux; S_h, area of the heater; R, radius of the heater; ε, admissible error.

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